A mathematical look to the world

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Abstract

When using visualisations to explore mathematics, I was seduced by their aesthetics. It led me to see them as a form of art and to start to produce mathematical visualisations just for the sake of it. I began to study the interactions between mathematics and art. At the same time, I discovered that mathematicians have a very special way to look to the world. This paper is an account of my research in the matter.

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1. Introduction

Most mathematicians present mathematics through symbols without any connection with reality. It is probably the reason why mathematics is often seen as useless, or worse. Nevertheless, the reality is the opposite: our entire world is full of mathematics, from rainbows, sand dunes, reflections on water, etc. to cellular phones and the structure of the World Wide Web (see [1]). But, to think that the world was created based on mathematical principles is a philosophical mistake. The truth is exactly the opposite: mathematical principles are the invention of the mathematicians, abstracted from their sensory experiences and their peculiar look to the world (see [5]), the subject of this article. So, it is important to get the abstract structures in mathematics linked to their concrete perceptive manifestations. That is my *credo* in mathematics and its teaching (see [2] and [3] for two examples, one at undergraduate level and the other at research level). This method led me to solve a conjecture by Douglas Hofstadter (see [4]), numerically first and then analytically. In addition, this method which I have used in math for a long time has also become a motivation for my art: photos, videos and digital paintings (see www.lehning.eu for examples of them). These digital techniques, combining reality and abstraction, can particularly help to promote this mathematical perception of reality.

2. A special look

As a mathematician, I realized that I do not take the same photographs as most people. Structures, patterns, curves, symmetries interest me more than anything. For example, Figure 1 is a shot I took in the Namib Desert (Namibia) because I was attracted by the strange pattern of the flowers (1 + 2 + 3) evoking the triangular numbers studied by the ancient Greek mathematicians.

Figure 1 Pattern of flowers

Figure 2 Parabola in dunes

Figure 2 was shot at the same place. It shows a parabola in the dunes. A lot of parabolas can be seen everywhere, especially on roofs to catch satellite TV and the Internet, or in the Himalaya (Nepal) to boil water (see Figure 3). Of course, the reason can be found in the mathematical definitions of parabolas and their astounding properties. These examples are a good introduction of it.

Figure 3 Parabola in the Himalaya

Figure 4 shows reflections on Gokyo lake in the Himalaya (Nepal), where the symmetry is almost perfect. Like this one, a lot of mathematical concepts invented by mathematicians already exist in nature or, more exactly, are abstracted from it (see [5]). We also meet symmetries in a more general sense like with the zebras and the kingfishers of Figures 5 and 6.

3. From art to mathematics

On one hand, mathematical structures are seen in nature, which does not mean that the world was created based on mathematical principles (see [5]). It just means that mathematicians are inspired by nature. Photographs of mathematical structures seen in nature can be illustrations of a mathematical theory or an incentive to study it. Perspectives, reflections or natural curves are good examples of that. On the other hand, art can be at the origin of mathematical studies. Symmetries, numbering or structures in paintings are good examples of that. The skin pattern of animals (studied by Alan Turing), as shown on Figure 5 is another example. The arrangement of the two birds on Figure 6 may lead to a more general sense of the notion of symmetry and to group theory.

Figure 4 Perfect symmetry

Figure 5 Zebras in symmetry

Figure 6 Kingfishers in symmetry

3.1. Art as an illustration

For example, we can introduce the intercept theorem (also called Thales' theorem from an ancient Greek mathematician named Thales of Miletus) with photographs of perspectives like the one on Figure 7, taken on the sea front of Saint Malo (France). With such photographs in my videos and lectures (see www.lehning.eu), I show their link with mathematics, in a poetic way and can also explain this link.

Figure 7 The intercept theorem on the shore

For that, we extend the lines of the timber piles (as AC and BD on Figure 8) and realize that they meet on the left, outside the photograph (at O on Figure 8). Thus, the intercept theorem comes into reality. The triangles OAB and OCD are similar, and we can do some computations too.

Figure 8 The theorem behind the sea front

We can also put a real parabola on the photograph of the dune (Figure 2). We obtain Figure 9 which is a good experimental evidence of the nature of the curve in the dune. Of course, even if the wind is the obvious cause, we need to study the origin of this parabolic form to understand it.

Figure 9 Apparition of the parabola

It works in a lot of cases. For example, in this way, we can recognize a circle behind a rainbow, circles in the Eiffel tower, ellipses in the stations of Paris underground, etc. It leads to properties of those curves… and to mathematical ideas to prove them.

Figure 10 A rainbow over the Victoria Falls

When we see a rainbow like the one on Figure 10, it is rather easy to think that it looks like a part of a circle. To make it obvious, we can draw a real circle and adjust it to fit the rainbow. On Figure 11, the point on the right is the centre of that circle. We can see that the circle centred there coincide with the rainbow. The study of the origin of a rainbow confirms that it is really a circle. It is the intersection of a plane and a cone with an opening angle of 84° and the line between the observer and the sun as the axis.

Figure 11 Circle coinciding with the rainbow

3.2. Art as a mathematical puzzle

For the mathematician, some abstract pieces of painting are mainly mathematical puzzles. For example, Figure 12 asks the question:

In how many ways can we divide a 3 by 3 square into 1, 3 and 5 adjacent squares?

A mathematical study shows that there are 10, not 9 as suggested by the painting.

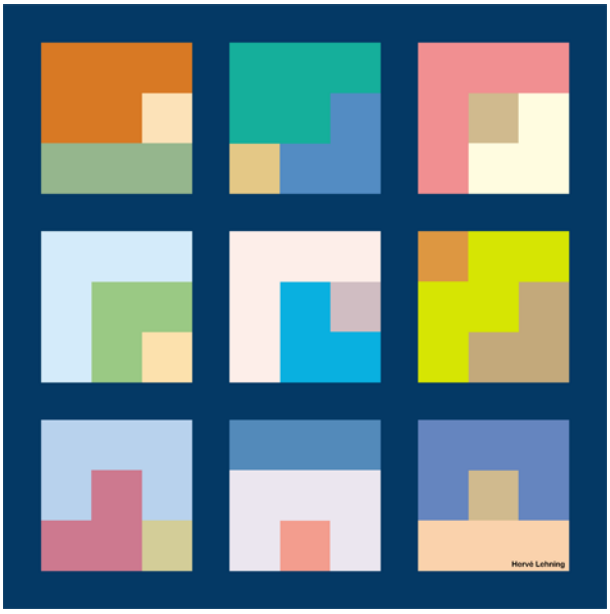

Figure 12 1 + 3 + 5 = 9

The answer is another piece of art, describing a way to tackle the question like a math article divided in 16 squares surrounding the main painting (see Figure 13).

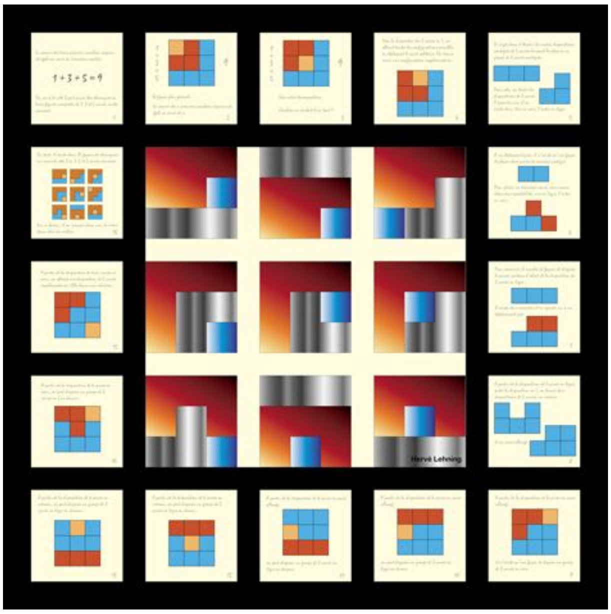

Figure 13 1 + 3 + 5 = 9 detailed

It begins with an analysis of the decomposition of a 3 by 3 square in 1, 3 and 5 adjacent squares. The simplest configuration is in the top left corner of Figure 14. By moving the pieces of the puzzles, we find two others possibilities (top right and bottom left).

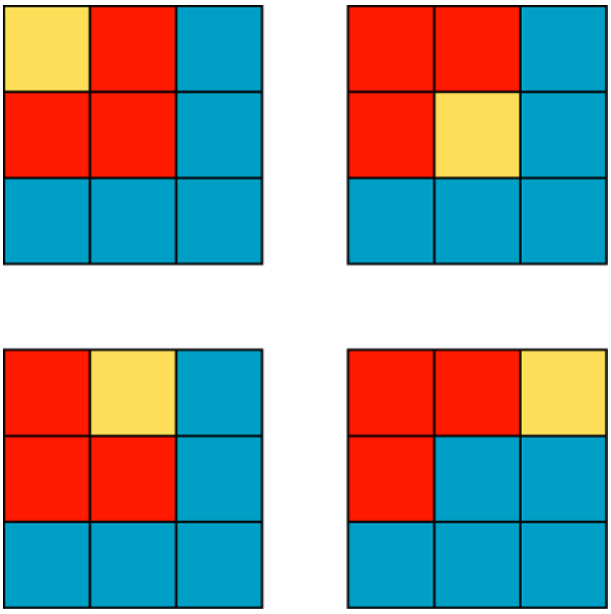

Figure 14 four possibilities

After that, we have to change the shape of the pieces. For example, the 3 squares piece takes two forms, a line and a corner. Moving this piece and 1 square leads to 10 possibilities (see Figure 13). One of them is not on Figure 12.

4. From mathematics to art

Some proofs without words can become pieces of art through visualisation. It is specially the case with the Pythagorean Theorem:

In any right-angled triangle, the area of the square whose side is the hypotenuse is equal to the sum of the areas of the squares whose sides are the two legs.

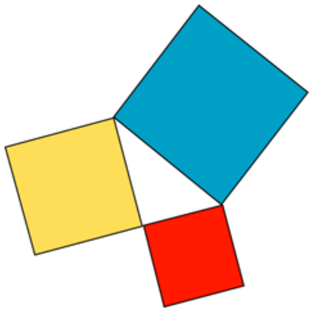

Figure 15 Pythagorean Theorem

That is to say: on Figure 15, the area of the big square is equal to the sum of the areas of the two smaller ones. One way to prove it is to divide the three squares in pieces and to move the pieces of the two small squares to build the big one.

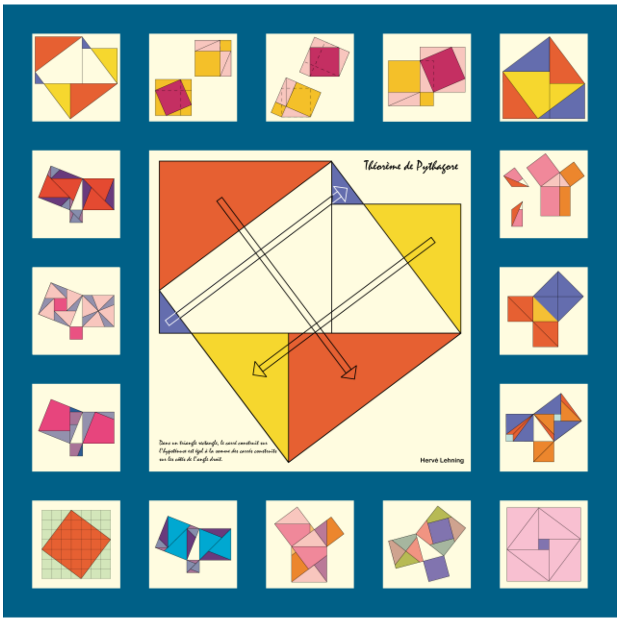

Figure 16 Pythagorean Theorem

Figure 16 offers 16 proofs of the Pythagorean Theorem. The proof in the central square is materialised by the arrows showing how to move the pieces to build the big square from the two small ones. Put together, these proofs without words form a piece of art. Thus, mathematics can be interpreted as art.

In 2000, an equation by Leonhard Euler, a mathematician of the XVIIIth century, was elected the most beautiful formula of the world by the readers of *The Mathematical Intelligencer*. Here is this formula:

$$e^{i\pi} + 1 = 0.$$

For a mathematician, the beauty of this formula comes from the joining of five of the most important mathematical constants in a very simple, but puzzling, equation, written with just an addition and an exponentiation. These constants are 0 and 1, the identities of addition and multiplication, i, the imaginary unit of complex numbers, e and π, the two most important transcendental numbers.

This choice can be summed up in this simple formula:

Depth + Simplicity = Beauty.

As a tribute to the beauty of Euler's formula, invented by the mathematical genius of the Enlightenment, I realised a lamp on the matter. It gives a proof without words of the formula. Creating beautiful objects is another way to popularise math through art (see Figure 17).

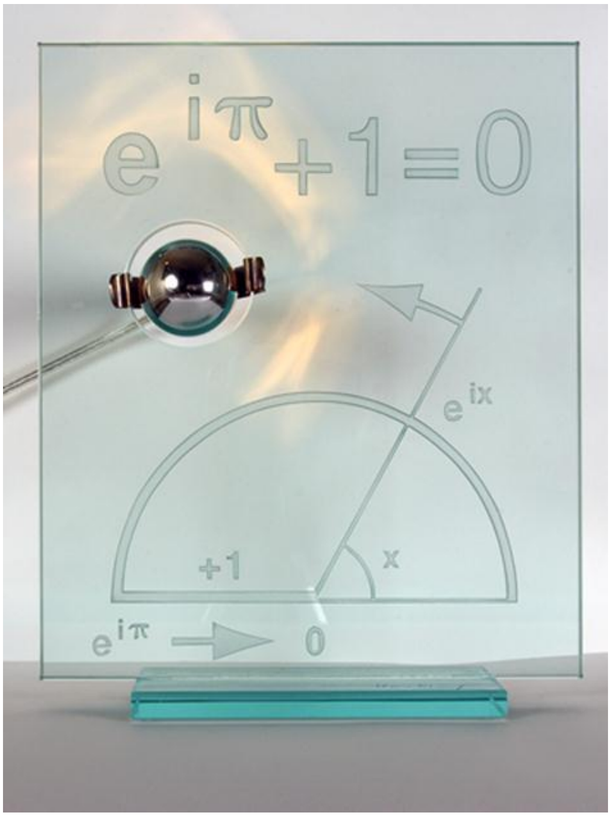

Figure 17 Euler's formula

Figure 18 The heart equation

At this occasion, I realized that the engraved glass is ideal to highlight equations, so I made several models of lamps like the one called "the heart equation" inspired by a curve named the cardioid.

5. Art evoking mathematics

For the mathematician, some structures seem to have a mathematical origin. It is the case for skin patterns (see Figure 5); but what are the mathematics behind it? The answer is not so easy. Photographs can be puzzling. In the case of Figure 19 showing an ice rock melting on a beach of Greenland, a poet may see two kissing birds but it could also evoke some topological questions to the mathematician.

Figure 19 Ice rock melting on a beach

Conclusions

In this paper, I described the uses of visualisations in mathematics and their aesthetic aspects. On one hand, visualisations are useful to understand mathematics better and to enhance our intuition of mathematical objects. On the other hand, the visualisations created are pieces of art that can popularise mathematics and show the beauty hidden behind mathematics.

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Credits

All the photographs, paintings and objects presented in this paper have been created by the author, Hervé Lehning.